butes to quadratic or linear behavior depends on the magnitude of  $\sigma_i/H$ , where  $\sigma_i$  is a measure of internal strain. If H is sufficiently large, this quantity will be small for all  $\sigma_i$  and quadratic behavior observed, while if H is sufficiently small, many local strain regions will contribute to linear behavior and the linear term will dominate.

(ii) Similar variation of a and b with increased internal strain, as observed by Parfenov and Voroshilov, is expected since local strain regions contributing to both linear and quadratic behavior would increase.

(iii) Parfenov and Voroshilov also observed that a is proportional to  $M_s$  under temperature variation in nickel. From this calculation, a is proportional to  $B/\mu M_s$  as shown by Eq. (13), and since, in nickel,  $B/\mu$  is proportional to  $M_{s}^{2,15}$  this behavior is expected.

## VII. CONCLUSION

The primary conclusion is to suggest that the a/H term in the expression for the approach to saturation has been overemphasized. Its origin is in the residual internal strain of magnetic material and it has validity only over a limited region of the H axis. Secondary results are the model and calculation which determine the magnetic behavior of porous magnetic material subject to hydrostatic pressure. It is worth mentioning that this technique suggests a method for controlled investigation of the effects of internal strain on material properties.

## ACKNOWLEDGMENT

The author wishes to thank Professor G. E. Duvall for his helpful discussions during the course of this work.

## APPENDIX

The following calculation will show that the subsequent functional dependence of  $M/M_s$  on P/H, after the initial quadratic behavior, is linear with a slope given by Eq. (13). The magnetic equilibrium relation, Eq. (9), can be written

$$\frac{r^3}{a^3} = \frac{3}{4} \frac{\sin 2(\psi + \theta)}{\sin \Psi} \frac{BP}{\mu M_s H} .$$
 (14)

This, in principle, can be solved for  $\cos\psi$ , giving

$$\cos\psi = g\left(\frac{u^3}{P/h}, \theta\right)$$
,

Park, Calif. 94025.

\*Research sponsored by the U. S. Air Force Office of Scientific Research under Contract No. AFOSR-69-1758. †Present address: Stanford Research Institute, Menlo where g is an unknown function, u = r/a, and  $h = \mu M_s H/B$  is the reduced field. Averaging  $\cos \psi$  over a spherical surface gives

$$(\cos\psi)_{av} = m(u) = k \left(\frac{u^3}{P/h}\right),$$

where k is another unknown function and m(u) is the average normalized magnetization in the direction of the applied field in a spherical shell at a radius r. This can be inverted to obtain

$$u^3 = (P/h)f(m)$$
 (15)

Again f is unknown. Equation (15) will be used in the following. First an expression for the macroscopic magnetization in the porous material is required. In terms of the proposed model in Sec. II this is

$$\frac{M}{M_s} = \frac{4\pi}{\frac{4}{3}\pi\gamma_0^3} \int_a^{r_0} m\gamma^2 d\gamma$$

or

$$\frac{M}{M_s} = 3p \int_1^{r_0/a} mu^2 du ,$$

where  $p = a^3 / r_0^3$  is porosity.

In anticipation of linear behavior consider

$$\frac{dM/M_s}{dP/h} = 3p \int_1^{r_0/a} \left. \frac{\partial m}{\partial P/h} \right|_u u^2 du .$$

The mathematical identity

$$\frac{\partial m}{\partial P/h}\Big|_{u} = -\frac{\partial u}{\partial P/h}\Big|_{m}\frac{\partial m}{\partial u}\Big|_{P/h}$$

with Eq. (15) gives

$$\frac{\partial m}{\partial P/h}\bigg|_{u} = -\frac{f(m)}{3u^2} \left.\frac{\partial m}{\partial u}\right|_{P/h} ,$$

and therefore

$$\frac{dM/M_s}{dP/h} = -p \int_1^{r_0/a} f(m) \left. \frac{\partial m}{\partial u} \right|_{P/h} du .$$

In a region where the magneto-elastic energy dominates at the lower integration limit while the magnetic energy dominates at the upper limit, the integral transforms to

$$\frac{dM/M_s}{dP/h} = -p \int_{\pi/4}^{-1} f(m) \, dm \, . \tag{16}$$

This shows the anticipated linear behavior which is expected to occur in some region of the P/h axis. Equation (16) is Eq. (13) with  $\gamma$  given by the integral expression.

<sup>1</sup>N. S. Akulov, Z. Physik <u>69</u>, 882 (1931).

<sup>2</sup>R. Becker and H. Polley, Ann. Physik <u>37</u>, 534 (1940). <sup>3</sup>W. E. Brown, Phys. Rev. <u>58</u>, 736 (1940); <u>60</u>, 132 (1941). <sup>4</sup>L. Néel, J. Phys. Radium <u>5</u>, 184 (1948).

<sup>5</sup>V. V. Parfenov and V. P. Voroshilov, Phys. Metals Metallog. (USSR) <u>13</u>, 340 (1962); <u>13</u>, 502 (1962).

<sup>6</sup>R. C. Wayne, G. A. Samara, and R. A. LeFever, J. Appl. Phys. <u>41</u>, 633 (1970).

<sup>7</sup>A. Buch and S. Goldschmidt, Materials Sci. Eng. <u>5</u>, 111 (1970).

<sup>8</sup>J. K. Mackenzie, Proc. Phys. Soc. (London) <u>62</u>, 2 (1950).

<sup>9</sup>J. Smit and H. P. J. Wijn, *Ferrites* (Wiley, New York, 1959), p. 119.

<sup>10</sup>C. Kittel, Rev. Mod. Phys. <u>21</u>, 541 (1949).

<sup>11</sup>This and later results are limiting cases of a general solution attributed to Lamé. See also L. O. Landau and E. M. Lifshitz, *Theory of Elasticity* (Addison-Wesley, Reading, Mass., 1955), p. 20.

<sup>12</sup>See, for instance, R. R. Birss, Proc. Phys. Soc. (London) <u>75</u>, 8 (1960).

<sup>13</sup>J. P. Hirth and J. Lothe, *Theory of Dislocations* (McGraw-Hill, New York, 1968).

<sup>14</sup>A. H. Morrish, *The Phsical Principal of Magnetism* (Wiley, New York, 1965), p. 395.

<sup>15</sup>E. W. Lee, Rept. Progr. Phys. <u>28</u>, 184 (1955).

The representation is to addred that the "Here is the account of the approximation address in the account of the approximation is in the rest of the account address in a system material and the rest of the rest of the rest material and the rest of the rest of the rest and the rest of the rest of the rest of the rest to the rest of the rest of the rest of the rest rest of the rest of the rest of the rest of the rest to the rest of the rest of the rest of the rest to the rest of the rest of the rest of the rest of the the rest of the rest of the rest of the rest of the the rest of the rest of the rest of the rest of the the rest of the the rest of the the rest of the r

TRUMPIC CONDER TH

\* The author wheteen small 23 min ever Q. D. During for his holpful in the space danie (the smileet the form).

- ATHING MADE

- All and the state of the solution of the the the solution of the solution

and a sub a sub

The in propriety gate by any other times in the

Linear and demonstration of Fills and Ale Parine Office and the Bline present of angles Continued to Alexandratic Trans Disease and a second of Second Statement Second Second States Calify Second Statement Statements

 Alexandria de la consulta de la consul

viewed to approve these said

and the first of the

In Arrowskiere for exciteta-elevent contegy dome trapes of the arrow preprintion (polls) who the rates actes entries doors are all set by or tracks, the arthread threads are so

This of an the solid road (need the second and in the second reaction of the second reaction of the Price area